Gauge-Yukawa Unification

in
$SO(10)$ SUSY GUTs

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Abstract

We study supersymmetric unified models with three fermion generations based on the gauge group $SO(10)$ and require Gauge-Yukawa Unification, i.e., a renormalization group invariant functional relationship among the gauge and Yukawa couplings of the third generation in the symmetric phase. In the case of the minimal model, we find that the predicted values for the top and bottom quark masses are in agreement with the present experimental data for a wide range of supersymmetry breaking scales. We also find that an experimental accuracy of less than 1% for the top quark mass could test the corresponding prediction of the Gauge-Yukawa unified model.

1 Introduction

The remarkable success of the standard model (SM) suggests that we have at hand a highly non-trivial part of a more fundamental theory for elementary particle physics. Since, however, the SM contains many independent parameters, it has been a challenge to understand the plethora of these free parameters. In Grand Unified Theories (GUTs) [1, 2], the gauge interactions of the SM are unified at a certain energy scale M_{GUT}, and consequently its gauge couplings are related with each other. Also the Yukawa couplings can be related among themselves to a certain extent. These relations among the couplings can yield testable predictions for GUTs [3, 4].

However, GUTs can not relate the gauge and Yukawa couplings with each other. In order to achieve Gauge-Yukawa Unification (GYU), within the assumption that all the particles appearing in a field theory model are elementary, one has to consider extended supersymmetry [5]. Unfortunately, it is extremely difficult to construct a realistic model based on the extended supersymmetry, because the model has a real structure with respect to $SU(2)_L \times U(1)_Y$ [5]. In superstrings and composite models such relations, in principle, also exist. However, in both cases there exist open difficult problems which among others are related to the lack of realistic models.

Recently, an alternative way to achieve unification of couplings [6]-[11] has been proposed; it is based on the fact that within the framework of a renormalizable field theory, one can find renormalization group (RG) invariant relations among parameters which can improve the calculability and the predictive power of the theory. This idea is called sometimes the principle of reduction couplings [6]. In this paper we would like to consider $SO(10)$ supersymmetric GUTs along the lines of this unification idea. We note that all realistic supersymmetric $SO(10)$ models have to be asymptotically-nonfree, because one needs a certain set of Higgses to break $SO(10)$ down to $SU(3)_C \times SU(2)_L \times U(1)_Y$ [2, 12].

The common wisdom is that the asymptotically-nonfree theories develop a Landau pole at a high energy scale, a fact which inevitably suggests that the theory is trivial, unless new physics is entering before the couplings blow up. However, there exist arguments leading to a different view point; the theory converges to a well-defined ultraviolet fixed

point, a "new phase", instead of blowing up [1]. Non-abelian gauge theories could have the same behavior, and it might be that an asymptotically nonfree, non-abelian gauge theory with matter couplings can change after the critical value of the couplings its phase due to a certain self-adjustment of the couplings and become a well defined finite theory [2]. In this way, a dynamical unification of couplings [17] [3] can be achieved, since to enter into the new phase the couplings are supposed to satisfy a definite relation. It is natural to assume that at scales below the critical value, that is, in the symmetric phase of a GUT these relations among the couplings are RG invariant as a remnant of the dynamical unification of couplings. When the GUT enters in its spontaneous broken phase, these RG invariant relations serve just as boundary conditions at M_{GUT} on the evolution of couplings for scales below it. Since the principle of reduction of couplings is based on RG invariant relations among couplings, the GYU based on this principle could be a consequence of the dynamical unification of couplings described above. This is a speculation, of course, because so far there exists no reliable and decisive calculation on the behavior of asymptotically-nonfree, non-abelian gauge theories [4].

There have been recently various phenomenological studies on $SO(10)$ supersymmetric GUTs without GYU [19]-[22], where the top quark mass was calculated from the requirement of the correct bottom-tau hierarchy (recall that the top Yukawa coupling contributes significantly to the RG evolution of the bottom and tau Yukawa couplings). Unfortunately, there exists a wide range of the predicted values of M_t ($160 - 200$ GeV [20]). One of the main reasons is that the bottom-tau hierarchy is experimentally known only with a large uncertainty ($\sim 10\%$ [23]). Suppose that the bottom-tau hierarchy would be precisely known and the calculated top mass would exactly agree with the experimental value. Then there should exist a unique Yukawa coupling of the third generation in

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[1]See ref. [13], which contain also earlier references on ultraviolet fixed points.

[2]Similar phenomenon has been observed in asymptotically free theories, in which the couplings have to be related with each other in order for the theories to be asymptotically free and hence well defined in the ultraviolet limit [8, 16].

[3]This analysis was motivated by the observation [18] that there might exist an infrared fixed point in asymptotically free QCD.

[4]Even the case of QED has not been completely clarified [14, 15].

$SO(10)$ GUTs, which is consistent with experimental data. It is, however, clear that this Yukawa coupling can not be predicted within the conventional GUT scheme.

With a GYU the model obtains a more predictive power, and the Yukawa couplings become calculable, as has been experienced in our recent studies on other models [9]-[11]. Although the GYU proposed there is a gradual, conservative extension of the usual GUT scheme, it has turned out to yield successful predictions. We emphasize that this success is not just a consequence of the infrared behavior of the Yukawa couplings [24]. Although the infrared behavior is an important ingredient for the successful predictions, we should stress also the significance of the field content of the theories as well as their interactions above the unification scale. The reason is that they contribute to the β functions in the symmetric phase which fix the structure of the GYU based on the principle of reduction of couplings and consequently the boundary conditions for the evolution of couplings below the GUT scale. Therefore, it is absolutely nontrivial that (1) there exist a unique Yukawa coupling of the third generation in a supersymmetric $SO(10)$ GUT that is consistent with the experimental data, and (2) this Yukawa coupling can be calculated by means of the reduction of couplings.

In this paper we will consider two different models; the first one with the Higgs super-multiplets of $\mathbf{1}, \mathbf{10}, \mathbf{16}, \overline{\mathbf{16}}, \mathbf{45}, \mathbf{54}$ and the three fermion generations in $\mathbf{16}$, and the second one with $\mathbf{126} + \overline{\mathbf{126}}$ instead of the singlet which provides the Georgi-Jarlskog mechanism [25, 12] to obtain a realistic mass fermion matrix. We have found that the first model yields experimentally consistent predictions, where we neglect the Yukawa coulings of the first two fermion generations. For the second model we obtain couplings which are so large that the model either cannot be treated within the framework of perturbation theory or does not give a testable prediction on the top-bottom hierarchy.

Our approach to predict the top and bottom quark masses, at first sight, looks similar to the infrared-fixed-point approach of ref. [26]. In the final section of this paper, we will discuss this approach within the framework of our concrete $SO(10)$ model and conclude that the infrared-fixed-point approach does not always provide us with precise predictions on low energy parameters.

2   The models

We denote the hermitean $SO(10)$-gamma matrices by Γ_α , $\alpha = 1, \cdots, 10$. The charge conjugation matrix C satisfies $C = C^{-1}$, $C^{-1}\Gamma_\alpha^T C = -\Gamma_\alpha$, and the Γ_{11} is defined as $\Gamma_{11} \equiv (-i)^5 \Pi_{\alpha=1}^{10}\Gamma_\alpha$ with $(\Gamma_{11})^2 = 1$. The chiral projection operators are given by $\mathcal{P}_\pm = \frac{1}{2}(1 \pm \Gamma_{11})$.

In $SO(10)$ GUTs [2, 12], three generations of quarks and leptons are accommodated by three chiral supermultiplets in **16** which we denote by

$$\Psi^I(\mathbf{16}) \text{ with } \mathcal{P}_+ \Psi^I = \Psi^I , \tag{1}$$

where I runs over the three generations and the spinor index is suppressed. To break $SO(10)$ down to $SU(3)_\mathrm{C} \times SU(2)_\mathrm{L} \times U(1)_\mathrm{Y}$, we use the following set of chiral superfields:

$$S_{\{\alpha\beta\}}(\mathbf{54}) , A_{[\alpha\beta]}(\mathbf{45}) , \phi(\mathbf{16}) , \overline{\phi}(\overline{\mathbf{16}}) . \tag{2}$$

The two $SU(2)_\mathrm{L}$ doublets which are responsible for the spontaneous symmetry breaking (SSB) of $SU(2)_\mathrm{L} \times U(1)_\mathrm{Y}$ down to $U(1)_\mathrm{EM}$ are contained in $H_\alpha(\mathbf{10})$.

2.1   Model I

For model I, we further introduce a singlet φ which after the SSB of $SO(10)$ will mix with the right-handed neutrinos so that they will become superheavy.

The superpotential of model I is given by

$$W^I = W_Y + W_{SB} + W_{HS} + W_{NM}^I + W_M , \tag{3}$$

where

$$
\begin{aligned}
W_Y &= \frac{1}{2}\sum_{I,J=1}^{3} g_{IJ}\,\Psi^I\,C\Gamma_\alpha\,\Psi^J\,H_\alpha , \\
W_{SB} &= \frac{g_\phi}{2}\,\overline{\phi}\,\Gamma_{[\alpha\beta]}\,\phi\,A_{[\alpha\beta]} + \frac{g_S}{3!}\,\mathrm{Tr}\,S^3 + \frac{g_A}{2}\,\mathrm{Tr}\,A^2\,S , \\
W_{HS} &= \frac{g_{HS}}{2}\,H_\alpha\,S_{\{\alpha\beta\}}\,H_\beta , \quad W_{NM}^I = \sum_{I=1}^{3} g_{INM}\,\Psi^I\,\overline{\phi}\,\varphi , \\
W_M &= \frac{m_H}{2}\,H^2 + m_\varphi\,\varphi^2 + m_\phi\,\overline{\phi}\phi + \frac{m_S}{2}\,S^2 + \frac{m_A}{2}\,A^2 ,
\end{aligned}
\tag{4}
$$

and $\Gamma_{[\alpha\beta]} = i(\Gamma_\alpha\Gamma_\beta - \Gamma_\beta\Gamma_\alpha)/2$. The superpotential is not the most general one, but by virtue of the non-renormalization theorem, this does not contradict the philosophy of the coupling unification by the reduction method (a RG invariant fine tuning is a solution of the reduction equation) . W_{SB} is responsible for the SSB of $SO(10)$ down to $SU(3)_C \times SU(2)_W \times U(1)_Y$, and this can be achieved without breaking supersymmetry, while W_{HS} is responsible for the triplet-doublet splitting of H. The right-handed neutrinos obtain a superheavy mass through W_{NM} after the SSB (as announced), and the Yukawa couplings for the leptons and quarks are contained in W_Y. We assume that there exists a choice of soft supersymmetry breaking terms so that all the vacuum expectation values necessary for the desired SSB corresponds to the minimum of the potential.

Given the supermultiplet content and the superpotential W, we can compute the β functions of the model. The gauge coupling of $SO(10)$ is denoted by g, and our normalization of the β functions is as usual, i.e., $dg_i/d\ln\mu = \beta_i^{(1)}/16\pi^2 + O(g^5)$, where μ is the renormalization scale. We find:

$$
\begin{aligned}
\beta_g^{(1)} &= 7\,g^3 \ , \\
\beta_{g_T}^{(1)} &= g_T\left(14|g_T|^2 + \frac{27}{5}|g_{HS}|^2 + |g_{3NM}|^2 - \frac{63}{2}g^2\right) , \\
\beta_{g_\phi}^{(1)} &= g_\phi\left(53|g_\phi|^2 + \frac{48}{5}|g_A|^2 + \frac{1}{2}|g_{1NM}|^2 + \frac{1}{2}|g_{2NM}|^2 + \frac{1}{2}|g_{3NM}|^2 - \frac{77}{2}g^2\right) , \\
\beta_S^{(1)} &= g_S\left(\frac{84}{5}|g_S|^2 + 12|g_A|^2 + \frac{3}{2}|g_{HS}|^2 - 60g^2\right) , \\
\beta_A^{(1)} &= g_A\left(16|g_\phi|^2 + \frac{28}{5}|g_S|^2 + \frac{116}{5}|g_A|^2 + \frac{1}{2}|g_{HS}|^2 - 52g^2\right) , \\
\beta_{HS}^{(1)} &= g_{HS}\left(8|g_T|^2 + \frac{28}{5}|g_S|^2 + 4|g_A|^2 + \frac{113}{10}|g_{HS}|^2 - 38g^2\right) , \qquad (5) \\
\beta_{1NM}^{(1)} &= g_{1NM}\left(\frac{45}{2}|g_\phi|^2 + 9|g_{1NM}|^2 + \frac{17}{2}|g_{2NM}|^2 + \frac{17}{2}|g_{3NM}|^2 - \frac{45}{2}g^2\right) , \\
\beta_{2NM}^{(1)} &= g_{2NM}\left(\frac{45}{2}|g_\phi|^2 + \frac{17}{2}|g_{1NM}|^2 + 9|g_{2NM}|^2 + \frac{17}{2}|g_{3NM}|^2 - \frac{45}{2}g^2\right) , \\
\beta_{3NM}^{(1)} &= g_{3NM}\left(5|g_T|^2 + \frac{45}{2}|g_\phi|^2 + \frac{17}{2}|g_{1NM}|^2 + \frac{17}{2}|g_{2NM}|^2 + 9|g_{3NM}|^2 - \frac{45}{2}g^2\right) .
\end{aligned}
$$

We have assumed that the Yukawa couplings g_{IJ} except for $g_T \equiv g_{33}$ vanish. They can be included as small perturbations [5]. Needless to say that the soft susy breaking terms do not alter the β functions above.

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[5]We will clarify later what we mean by small perturbations.

2.2   Model II

For model II, we introduce a pair of

$$\Theta_{[\alpha\beta\gamma\mu\nu]}(\mathbf{126}) \ \text{ and } \ \overline{\Theta}_{[\alpha\beta\gamma\mu\nu]}(\overline{\mathbf{126}}) \tag{6}$$

instead of the singlet φ, providing us with a possibility of incorporating the Georgi-Jarlskog mechanism [25, 12]. They satisfy the duality conditions

$$\Theta_{[\alpha_1\cdots\alpha_5]} \ (\overline{\Theta}_{[\alpha_1\cdots\alpha_5]}) \ = \ -(+)\frac{i}{5!}\epsilon_{\alpha_1\cdots\alpha_{10}} \ \Theta_{[\alpha_6\cdots\alpha_{10}]} \ (\overline{\Theta}_{[\alpha_6\cdots\alpha_{10}]}) \ , \tag{7}$$

and $\overline{\Theta}$ (instead of φ) also will mix with the right-handed neutrinos to make them super-heavy.

The superpotential of model II is given by

$$W^{II} \ = \ W_Y + W_{SB} + W_{HS} + W_\Theta + W_{GJ} + W_{NM}^{II} + W_M \ , \tag{8}$$

where

$$\begin{aligned}
W_\Theta \ &= \ \frac{g_{\Theta S}}{4!8} \ \Theta_{[\alpha_1\cdots\alpha_5]} \ S_{\{\alpha_5\beta_1\}} \ \Theta_{[\beta_1\cdots\beta_5]} + \frac{g_{\overline{\Theta}S}}{4!8} \ \overline{\Theta}_{[\alpha_1\cdots\alpha_5]} \ S_{\{\alpha_5\beta_1\}} \ \overline{\Theta}_{[\beta_1\cdots\beta_5]} \\
&\quad + \frac{g_{\Theta A}}{4!4} \ \overline{\Theta}_{[\alpha_1\cdots\alpha_5]} \ A_{[\alpha_5\beta_1]} \ \Theta_{[\beta_1\cdots\beta_5]} \ , \\
W_{GJ} \ &= \ \frac{g_{GJ}}{5!4} \ \Psi^2 \, C\Gamma_{[\alpha_1\cdots\alpha_5]} \ \Psi^2 \ \overline{\Theta}_{[\alpha_1\cdots\alpha_5]} \ , \\
W_{NM}^{II} \ &= \ \frac{1}{5!2} \sum_{I=1}^{3} g_{INM} \ \Psi^I \, C\Gamma_{[\alpha_1\cdots\alpha_5]} \ \phi \ \overline{\Theta}_{[\alpha_1\cdots\alpha_5]} \ ,
\end{aligned}$$

and $\Gamma_{[\alpha_1\cdots\alpha_5]} = \frac{1}{5!}\,(\Gamma_{\alpha_1}\cdots\Gamma_{\alpha_5} +$ anti-symmetric permutations). (W_Y , W_{SB} , W_{HS} and W_M are given in eq. (4), and W_M contains the $\Theta - \overline{\Theta}$ mass term instead of the φ mass term.) The β functions are found to be

$$\begin{aligned}
\beta_g^{(1)} \ &= \ 77\,g^3 \ , \\
\beta_{g_T}^{(1)} \ &= \ g_T \left(14|g_T|^2 + \frac{27}{5}|g_{HS}|^2 + 126|g_{3NM}|^2 - \frac{63}{2}g^2 \right) , \\
\beta_{g_\phi}^{(1)} \ &= \ g_\phi \big(53|g_\phi|^2 + \frac{48}{5}|g_A|^2 + 63|g_{1NM}|^2 + 63|g_{2NM}|^2 + 63|g_{3NM}|^2 + \frac{35}{4}|g_{\Theta A}|^2 - \frac{77}{2}g^2 \big) , \\
\beta_S^{(1)} \ &= \ g_S \big(\frac{84}{5}|g_S|^2 + 12|g_A|^2 + \frac{3}{2}|g_{HS}|^2 + \frac{105}{8}|g_{\Theta S}|^2 + \frac{105}{8}|g_{\overline{\Theta}S}|^2 - 60g^2 \big) , \\
\beta_A^{(1)} \ &= \ g_A \big(16|g_\phi|^2 + \frac{28}{5}|g_S|^2 + \frac{116}{5}|g_A|^2 + \frac{1}{2}|g_{HS}|^2 + \frac{35}{2}|g_{\Theta A}|^2 + \frac{35}{8}|g_{\overline{\Theta}S}|^2
\end{aligned}$$

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$$+\frac{35}{8}|g_{\Theta S}|^2 - 52g^2\,) \, ,$$

$$\beta_{HS}^{(1)} = g_{HS}(\, 8|g_T|^2 + \frac{28}{5}|g_S|^2 + 4|g_A|^2 + \frac{113}{10}|g_{HS}|^2 + \frac{35}{8}|g_{\overline{\Theta}S}|^2 + \frac{35}{8}|g_{\Theta S}|^2 - 38g^2\,) \, , \qquad (9)$$

$$\beta_{1NM}^{(1)} = g_{1NM}(\, \frac{45}{2}|g_\phi|^2 + 134|g_{1NM}|^2 + 71|g_{2NM}|^2 + 71|g_{3NM}|^2 + 4|g_{GJ}|^2$$
$$+\frac{25}{8}|g_{\Theta A}|^2 + 25|g_{\overline{\Theta}S}|^2 - \frac{95}{2}g^2\,) \, ,$$

$$\beta_{2NM}^{(1)} = g_{2NM}(\, \frac{45}{2}|g_\phi|^2 + 71|g_{1NM}|^2 + 134|g_{2NM}|^2 + 71|g_{3NM}|^2 + 67|g_{GJ}|^2$$
$$+\frac{25}{8}|g_{\Theta A}|^2 + 25|g_{\overline{\Theta}S}|^2 - \frac{95}{2}g^2\,) \, ,$$

$$\beta_{3NM}^{(1)} = g_{3NM}(\, 5|g_T|^2 + \frac{45}{2}|g_\phi|^2 + 71|g_{1NM}|^2 + 71|g_{2NM}|^2 + 134|g_{3NM}|^2$$
$$+4|g_{GJ}|^2 + \frac{25}{8}|g_{\Theta A}|^2 + 25|g_{\overline{\Theta}S}|^2 - \frac{95}{2}g^2\,) \, ,$$

$$\beta_{\Theta S}^{(1)} = g_{\Theta S}(\, \frac{28}{5}|g_S|^2 + 4|g_A|^2 + \frac{1}{2}|g_{HS}|^2 + \frac{25}{4}|g_{\Theta A}|^2 + \frac{35}{8}|g_{\overline{\Theta}S}|^2$$
$$+\frac{85}{8}|g_{\Theta S}|^2 - 70g^2\,) \, ,$$

$$\beta_{\overline{\Theta}S}^{(1)} = g_{\overline{\Theta}S}(\, \frac{28}{5}|g_S|^2 + 4|g_A|^2 + \frac{1}{2}|g_{HS}|^2 + 8|g_{GJ}|^2 + 16|g_{1NM}|^2$$
$$+16|g_{2NM}|^2 + 16|g_{3NM}|^2 + \frac{25}{4}|g_{\Theta A}|^2 + \frac{435}{8}|g_{\overline{\Theta}S}|^2 + \frac{35}{8}|g_{\Theta S}|^2 - 70g^2\,) \, ,$$

$$\beta_{\Theta A}^{(1)} = g_{\Theta A}(\, \frac{48}{5}|g_A|^2 + 8|g_\phi|^2 + 4|g_{GJ}|^2 + 8|g_{1NM}|^2 + 8|g_{2NM}|^2 + 8|g_{3NM}|^2$$
$$+15|g_{\Theta A}|^2 + 25|g_{\overline{\Theta}S}|^2 + \frac{25}{8}|g_{\Theta S}|^2 - 66g^2\,) \, ,$$

$$\beta_{GJ}^{(1)} = g_{GJ}(\, 130|g_{GJ}|^2 + 8|g_{1NM}|^2 + 134|g_{2NM}|^2 + 8|g_{3NM}|^2 + \frac{25}{8}|g_{\Theta A}|^2 + 25|g_{\overline{\Theta}S}|^2 - \frac{95}{2}g^2\,) \, .$$

Observe the occurrence of large coefficients in the β functions above. They are responsible for the fact that the model II either cannot be treated in perturbation theory or does not give a testable prediction on the top quark mass, as we will see.

3   Gauge-Yukawa Unification

The principle of reduction of coupling is to impose as many as possible RG invariant constraints which are compatible with renormalizability [6]. Such constraints in the space of couplings can be expressed in the implicit form as $\Phi(g_1, \cdots, g_N) = $ const., which has

to satisfy the partial differential equation

$$\vec{\beta} \cdot \vec{\nabla} \, \Phi \;\; = \;\; \sum_{i=0}^{N} \beta_i \frac{\partial}{\partial g_i} \, \Phi \;\; = \;\; 0 \; , \tag{10}$$

where β_i is the β function of g_i. In general, there exist, at least locally, N indepen-
dent solutions of (10), and they are equivalent to the solutions of the so-called reduction
equations [6],

$$\beta \frac{dg_i}{dg} \;\; = \;\; \beta_i \; , \;\; i = 1, \cdots, N \; , \tag{11}$$

where $g \equiv g_0$ and $\beta \equiv \beta_0$. Since maximally N independent RG invariant constraints in
the $(N+1)$-dimensional space of couplings can be imposed by Φ_i, one could in principle
express all the couplings in terms of a single coupling, the primary coupling g [6]. This
possibility is without any doubt attractive, but it can be unrealistic. Therefore, one
often would like to impose fewer RG invariant constraints, leading to the idea of partial
reduction [7, 8].

Here we would like to briefly outline the method [6]. For the case at hand, it is convenient
to work with the absolute square of g_i, and we define the tilde couplings by

$$\tilde{\alpha}_i \;\; \equiv \;\; \frac{\alpha_i}{\alpha} \; , \;\; i = 1, \cdots, N \; ,$$

where $\alpha \; = \; |g|^2/4\pi$ and $\alpha_i \; = \; |g_i|^2/4\pi$. We assume that their evolution equations take
the form

$$\frac{d\alpha}{dt} \;\; = \;\; -b^{(1)} \, \alpha^2 + \cdots \; ,$$
$$\frac{d\alpha_i}{dt} \;\; = \;\; -b_i^{(1)} \, \alpha_i \alpha + \sum_{j,k} b_{i,jk}^{(1)} \, \alpha_j \alpha_k + \cdots \; , \tag{12}$$

in perturbation theory, and then we derive from (12)

$$\alpha \frac{d\tilde{\alpha}_i}{d\alpha} \;\; = \;\; \left(-1 + \frac{b_i^{(1)}}{b^{(1)}} \right) \tilde{\alpha}_i - \sum_{j,k} \frac{b_{i,jk}^{(1)}}{b^{(1)}} \, \tilde{\alpha}_j \, \tilde{\alpha}_k + \sum_{r=2} \left(\frac{\alpha}{\pi} \right)^{r-1} \tilde{b}_i^{(r)}(\tilde{\alpha}) \; , \tag{13}$$

where $\tilde{b}_i^{(r)}(\tilde{\alpha})$, $r = 2, \cdots$, are power series of $\tilde{\alpha}_i$ and can be computed from the r-th loop
β functions. We then solve the algebraic equations

$$\left(-1 + \frac{b_i^{(1)}}{b^{(1)}} \right) \rho_i - \sum_{j,k} \frac{b_{i,jk}^{(1)}}{b^{(1)}} \, \rho_j \, \rho_k \;\; = \;\; 0 \; , \tag{14}$$

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[6] Detailed discussions on partial reduction are given in ref. [10], for instance.

which give the fixed points of (13) at $\alpha = 0$. We assume that the solutions ρ_i's have the form

$$\rho_i \;=\; 0 \text{ for } i = 1, \cdots, N' \; ; \; \rho_i \;>\; 0 \text{ for } i = N'+1, \cdots, N \;, \tag{15}$$

and we regard $\tilde{\alpha}_i$ with $i \leq N'$ as small perturbations to the undisturbed system which is defined by setting $\tilde{\alpha}_i$ with $i \leq N'$ equal to zero. It is possible [6] to verify at the one-loop level the existence of the unique power series solutions

$$\tilde{\alpha}_i \;=\; \rho_i + \sum_{r=2} \rho_i^{(r)} \left(\frac{\alpha}{\pi}\right)^{r-1} \;,\; i = N'+1, \cdots, N \tag{16}$$

of the reduction equations (13) to all orders in the undisturbed system (as we will demonstrate it in our $SO(10)$ model below). These are RG invariant relations among couplings that keep formally perturbative renormalizability of the undisturbed system. So in the undisturbed system there is only *one independent* coupling α.

We emphasize that the more vanishing ρ_i's a solution contains, the less is its predictive power in general. We therefore search for predictive solutions in a systematic fashion.

3.1 Unperturbed system

(a) Model I

We find that for model I there exist two independent solutions, A and B, that have the most predictive power, where we have chosen the $SO(10)$ gauge coupling as the primary coupling:

$$
\rho_T = \begin{cases} 163/60 & \simeq 2.717 \\ 0 \end{cases} , \quad
\rho_\phi = \begin{cases} 5351/9180 & \simeq 0.583 \\ 1589/2727 & \simeq 0.583 \end{cases} ,
$$

$$
\rho_S = \begin{cases} 152335/51408 & \simeq 2.963 \\ 850135/305424 & \simeq 2.783 \end{cases} , \quad
\rho_A = \begin{cases} 31373/22032 & \simeq 1.424 \\ 186415/130896 & \simeq 1.424 \end{cases} ,
$$

$$
\rho_{HS} = \begin{cases} 7/81 & \simeq 0.086 \\ 170/81 & \simeq 2.099 \end{cases} , \quad
\rho_{1NM} = \rho_{2NM} = \begin{cases} 191/204 & \simeq 0.936 \\ 191/303 & \simeq 0.630 \end{cases} ,
$$

$$
\rho_{3NM} = \begin{cases} 0 \\ 191/303 & \simeq 0.630 \end{cases} \text{ for } \begin{cases} IA \\ IB \end{cases} . \tag{17}
$$

Clearly, the solution B has less predictive power because $\rho_T = 0$. So, we consider below only the solution A, in which the coupling α_{3NM} should be regarded as a small perturbation because $\rho_{3NM} = 0$.

Given this solution, we would like to show next (as promised) that the expansion coefficients $\rho_i^{(r)}$, $i = T, \cdots, 2NM$ can be uniquely computed in any finite order in perturbation theory. To this end, we assume that $\rho_i^{(n)}$ with $r \leq n-2$ are known, then insert the power series ansatz (16) for $\rho_i^{(n)}$, $i \neq 3NM$ into the reduction equation (13) and collect terms of $O(\alpha^{n-1})$. One finds easily that

$$\sum_{j \neq 3NM} M_{i\,j}(n)\, \rho_j^{(n)} = \quad \text{known quantities by assumption}, \ i \neq 3NM, \qquad (18)$$

where

$$M(n) = \begin{pmatrix} 1141/30 - 7n & 0 & 0 & 0 \\ 0 & 283603/9180 - 7n & 0 & 21404/3825 \\ 0 & 0 & 30467/612 - 7n & 152335/4284 \\ 0 & 31373/1377 & 219611/27540 & 909817/27540 - 7n \\ 56/81 & 0 & 196/405 & 28/81 \\ 0 & 2865/136 & 0 & 0 \\ 0 & 2865/136 & 0 & 0 \end{pmatrix}$$

$$\begin{matrix} 1467/100 & 0 & 0 \\ 0 & 5351/18360 & 5351/18360 \\ 152335/34272 & 0 & 0 \\ 31373/44064 & 0 & 0 \\ -791/810 - 7n & 0 & 0 \\ 0 & 573/68 - 7n & 191/24 \\ 0 & 191/24 & 573/68 - 7n \end{matrix} \qquad (19)$$

So, if $\det M(n) \neq 0$, the coefficients $\rho_i^{(n)}$, $i = T, \cdots, 2NM$ can be uniquely calculated. We in fact find

$$\det M(n) = -\frac{11092023863500 3554634381}{8522204882112000} + n\frac{36088745673180 92545318601}{25566614646336000}$$
$$+n^2\frac{757110512248666 9715209741}{8522204882112000} - n^3\frac{391617250274453 557751579}{284073496070400}$$

$$+n^4\frac{598654192729460650727}{819127728000} - n^5\frac{107001680791190563}{606761280}$$
$$+n^6\frac{108620968687}{5508} - n^7 823543$$
$$\neq \quad 0 \quad \text{for integer } n .\tag{20}$$

Therefore, there exists a unique power series solution of the form (16) for the solution IA.

(b) Model II

According to the principle of reduction of couplings, we search for most predictive solutions of (14). Of these solutions, we consider only non-degenerate ones with $\rho_T \neq 0$, because they are more predictive. We find that there exist three solutions, IIA, IIB and IIC, and they contain three vanishing ρ's:

$$\rho_T = \begin{cases} \frac{674137}{117840} & \simeq 5.7 \\ \frac{108764}{14225} & \simeq 7.7 \\ \frac{443}{60} & \simeq 7.4 \end{cases} , \quad \rho_{GJ} = \begin{cases} \frac{674137}{2969568} & \simeq 0.2 \\ \frac{390223}{1433880} & \simeq 0.3 \\ \frac{443}{1512} & \simeq 0.3 \end{cases} , \quad \rho_\phi = \begin{cases} \frac{674137}{1060560} & \simeq 0.6 \\ \frac{300557}{512100} & \simeq 0.6 \\ \frac{443}{540} & \simeq 0.8 \end{cases} ,$$

$$\rho_S = \begin{cases} 0 \\ 0 \\ \frac{16361}{4032} \simeq 4.1 \end{cases} , \quad \rho_A = \begin{cases} \frac{2356151}{5090688} \simeq 0.5 \\ 0 \\ \frac{4739}{1296} \simeq 3. \end{cases} , \quad \rho_{HS} = \begin{cases} \frac{1673861}{318168} \simeq 5.3 \\ \frac{13811}{51210} \simeq 0.3 \\ \frac{77}{81} \simeq 1.0 \end{cases} ,$$

$$\rho_{\Theta S} = \begin{cases} 0 \\ \frac{47260924}{4480875} \simeq 10.5 \\ 0 \end{cases} , \quad \rho_{\overline{\Theta}S} = \begin{cases} \frac{673145}{371196} \simeq 1.8 \\ \frac{1584242}{1493625} \simeq 1.1 \\ \frac{16937}{9450} \simeq 1.8 \end{cases} , \quad \rho_{\Theta A} = \begin{cases} \frac{205226}{36825} \simeq 5.6 \\ \frac{21623024}{4480875} \simeq 4.8 \\ 0 \end{cases} ,$$

$$\rho_{1NM} = \begin{cases} \frac{674137}{2969568} \simeq 0.2 \\ \frac{53321}{159320} \simeq 0.3 \\ \frac{443}{1512} \simeq 0.3 \end{cases} , \quad \rho_{2NM} = \begin{cases} \frac{674137}{2969568} \simeq 0.2 \\ \frac{53321}{159320} \simeq 0.3 \\ \frac{443}{1512} \simeq 0.3 \end{cases} ,$$

$$\rho_{3NM} = \begin{cases} 0 \\ 0 \\ 0 \end{cases} \quad \text{for} \begin{cases} \text{IIA} \\ \text{IIB} \\ \text{IIC} \end{cases} .\tag{21}$$

Observe that certain ρ's for solutions IIB and IIC are so large that the model cannot be treated in perturbation theory. The $\rho_T \simeq 5.7$ for solution IIA could be within the regime of perturbation theory, but as we will see in the next section, that value is so large that

the predicted value of M_t cannot be distinguished from its infrared value. Therefore, this model does not yield a testable prediction on the top quark mass.

We presented in this section the negative result, too, in some detail to emphasize that the existence of a consistent supersymmetric Gauge-Yukawa unified model based on $SO(10)$ is a nontrivial matter, as we have announced in the introduction.

3.2 Small perturbations

The small perturbations caused by nonvanishing $\tilde{\alpha}_i$ with $i \leq N'$, defined in eq. (15) and $\tilde{\alpha}_{3NM}$ in the case of solution IA, enter in such a way that the reduced couplings, i.e., $\tilde{\alpha}_i$ with $i > N'$, become functions not only of α but also of $\tilde{\alpha}_i$ with $i \leq N'$. It turned out [8] that, to investigate such partially reduced systems, it is most convenient to work with the partial differential equations, which for solution IA are

$$\left\{ \ \tilde{\beta}\frac{\partial}{\partial\alpha} + \beta_{3NM}\frac{\partial}{\partial\tilde{\alpha}_{3NM}} \ \right\} \tilde{\alpha}_i(\alpha,\tilde{\alpha}) \ = \ \tilde{\beta}_i(\alpha,\tilde{\alpha}) \ , \ i \neq 3NM \ , \tag{22}$$

where

$$\tilde{\beta}_i \ = \ \frac{\beta_i}{\alpha^2} - \frac{\beta}{\alpha^2}\,\tilde{\alpha}_i \ , \ \tilde{\beta} \ = \ \frac{\beta}{\alpha} \ .$$

These partial differential equations are equivalent to the reduction equations (13), and we look for solutions of the form

$$\tilde{\alpha}_i \ = \ \rho_i + \sum_{r=2}\left(\frac{\alpha}{\pi}\right)^{r-1} f_i^{(r)}(\tilde{\alpha}_{3NM}) \ , \ i \neq 3NM \ , \tag{23}$$

where $f_i^{(r)}(\tilde{\alpha}_{3NM})$ are supposed to be power series of $\tilde{\alpha}_{3NM}$. This particular type of solution can be motivated by requiring that in the limit of vanishing perturbations we obtain the undisturbed solutions (16) [8], i.e., $f_i^{(r)}(0) = \rho_i^{(r)}$ for $r \geq 2$. Again it is possible to obtain the sufficient conditions for the uniqueness of $f_i^{(r)}$ in terms of the lowest order coefficients. The proof is similar to that for $\rho_i^{(r)}$'s.

We have computed these corrections up to and including terms of $O(\tilde{\alpha}_{3NM}^2)$:

$$\tilde{\alpha}_T \ = \ (\,163/60 - 0.108\cdots\tilde{\alpha}_{3NM} + 0.482\cdots\tilde{\alpha}_{3NM}^2 + \cdots) + \cdots \ ,$$

$$\tilde{\alpha}_\phi \ = \ (\,5351/9180 + 0.316\cdots\tilde{\alpha}_{3NM} + 0.857\cdots\tilde{\alpha}_{3NM}^2 + \cdots) + \cdots \ ,$$
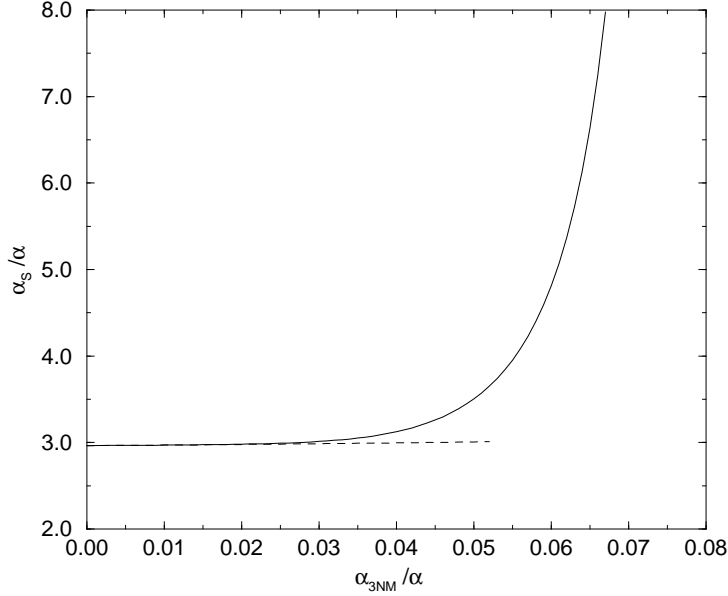
Figure 1: $\tilde{\alpha}_S$ versus $\tilde{\alpha}_{3NM}$, where the dashed line is obtained from the analytic expression (24).

$$
\begin{aligned}
\tilde{\alpha}_S &= (\,152335/51408 + 0.573\cdots\tilde{\alpha}_{3NM} + 5.7504\cdots\tilde{\alpha}_{3NM}^2 + \cdots\,) + \cdots \;, \\
\tilde{\alpha}_A &= (\,31373/22032 - 0.591\cdots\tilde{\alpha}_{3NM} - 4.832\cdots\tilde{\alpha}_{3NM}^2 + \cdots\,) + \cdots \;, \qquad (24) \\
\tilde{\alpha}_{HS} &= (7/81 - 0.00017\cdots\tilde{\alpha}_{3NM} + 0.056\cdots\tilde{\alpha}_{3NM}^2 + \cdots\,) + \cdots \;, \\
\tilde{\alpha}_{1NM} &= \tilde{\alpha}_{2NM} = (\,191/204 - 4.473\cdots\tilde{\alpha}_{3NM} + 2.831\cdots\tilde{\alpha}_{3NM}^2 + \cdots\,) + \cdots \;,
\end{aligned}
$$

where $\cdots$ indicates higher order terms which can be uniquely computed. In the partially reduced theory defined above, we have two independent couplings, α and α_{3NM} (along with the Yukawa couplings α_{IJ} , $I, J \neq T$).

At the one-loop level eq. (24) defines a line parametrized by $\tilde{\alpha}_{3NM}$ in the 7 dimensional space of couplings. A numerical analysis shows that this line blows up in the direction of $\tilde{\alpha}_S$ at a finite value of $\tilde{\alpha}_{3NM}$. Fig. 1 shows $\tilde{\alpha}_S$ as a function of $\tilde{\alpha}_{3NM}$ (the dashed line is obtained from the analytic expression (24)). So if we require $\tilde{\alpha}_S$ to remain within the perturbative regime (i.e., $g_S \lesssim 2$, which means $\tilde{\alpha}_S \lesssim 8$ because $\alpha_{\mathrm{GUT}} \sim 0.04$), the $\tilde{\alpha}_{3NM}$ should be restricted to be below ~ 0.067. As a consequence, the value of $\tilde{\alpha}_T$ is also bounded. To see this, we plot $\tilde{\alpha}_T$ as a function of $\tilde{\alpha}_{3NM}$ in fig. 2, from which we conclude
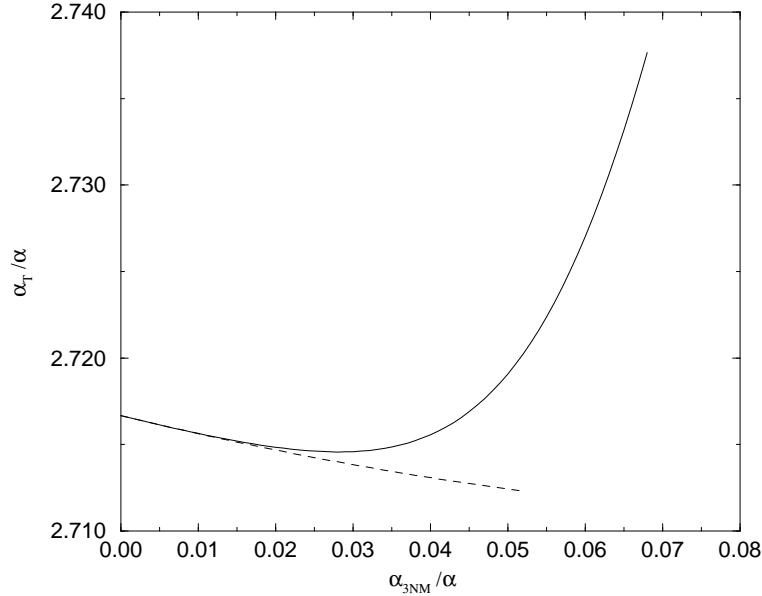
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Figure 2: $\tilde{\alpha}_T$ versus $\tilde{\alpha}_{3NM}$, where the dashed line is obtained from the analytic expression (24).

that

$$2.714 \;\lesssim\; \tilde{\alpha}_T \lesssim 2.736 \;. \tag{25}$$

This defines GYU boundary conditions holding at the unification scale M_{GUT} in addition to the group theoretic one, $\alpha_T = \alpha_t = \alpha_b = \alpha_\tau$. The value of $\tilde{\alpha}_T$ is practically fixed so that in the following discussions we may assume that $\tilde{\alpha}_T = 163/60 \simeq 2.72$, which is the unperturbed value.

4 Predictions

As pointed out, the GYU conditions (25) we have obtained above remain unaffected by soft supersymmetry breaking terms, because the β functions are not altered by these terms. To predict observable parameters from GYU, we apply the renormalization group technique [27, 22].

Just below M_{GUT} we would like to obtain the MSSM while requiring that all the

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superpartners are decoupled below the supersymmetry breaking scale M_{SUSY}. To simplify our numerical analysis we assume a unique threshold M_{SUSY} for all the superpartners. Then the SM should be spontaneously broken down to $SU(3)_C \times U(1)_{EM}$ due to the vacuum expectation value of the scalar component of H_α. We also assume that the low energy theory which satisfies the requirement above can be obtained by arranging soft supersymmetry breaking terms and the mass parameters in the superpotential (3) in an appropriate fashion.

We shall examine numerically the evolution of the gauge and Yukawa couplings including the two-loop effects, according to their renormalization group equations [7]. The translation of the value of a Yukawa coupling into the corresponding mass value follows according to $m_i = g_i(\mu)\,v(\mu)/\sqrt{2}$, $i = t, b, \tau$, where $m_i(\mu)$'s are the running masses and we use $v(M_Z) = 246.22$ GeV. The pole mass M_i can be calculated from the running one, and for the top mass, we use [22]

$$M_t \;=\; m_t(M_t)\,[\,1 + \frac{4}{3}\frac{\alpha_3(M_t)}{\pi} + 10.95\,(\frac{\alpha_3(M_t)}{\pi})^2\,]\;,\qquad (26)$$

where we compute $v(M_t)$ from $v(M_Z)$. As for the tau and bottom masses, we assume that $m_\tau(\mu)$ and $m_b(\mu)$ for $\mu < M_Z$ satisfy the evolution equation governed by the $SU(3)_C \times U(1)_{\text{EM}}$ theory with five flavors and use [22]

$$M_b \;=\; m_b(M_b)\,[\,1 + \frac{4}{3}\frac{\alpha_{3(5f)}(M_b)}{\pi} + 12.4\,(\frac{\alpha_{3(5f)}(M_b)}{\pi})^2\,]\;,$$
$$M_\tau \;=\; m_\tau(M_\tau)\,[\,1 + \frac{\alpha_{\text{EM}(5f)}(M_\tau)}{\pi}\,]\;,\qquad (27)$$

where the couplings with five flavors $\alpha_{3(5f)}$ and $\alpha_{\text{EM}(5f)}$ are related to α_3 and α_{EM} by

$$\alpha_{3(5f)}^{-1}(M_Z) \;=\; \alpha_3^{-1}(M_Z) - \frac{1}{3\pi}\frac{M_t}{M_Z}\;,\qquad (28)$$

$$\alpha_{\text{EM}(5f)}^{-1}(M_Z) \;=\; \alpha_{\text{EM}}^{-1}(M_Z) - \frac{8}{9\pi}\frac{M_t}{M_Z}\;.\qquad (29)$$

The corrections in eq. (27) are the SM ones, and in general one should add the MSSM corrections too. They could be even large, especially for $M_b \sim \pm O(20 - 30\%)$, in the case of universal soft supersymmetry breaking terms, while they can be kept small if

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[7]We take into account the threshold effects by the step function approximation of the β functions.

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these terms are not universal [20, 21]. As we will see below, our prediction for $m_b(M_b)$ without the MSSM corrections fit to the experimental value so that the model favors the non-universal soft supersymmetry breaking terms.

Regarding now

$$
\begin{aligned}
M_\tau &= 1.777 \text{ GeV} , \ M_Z = 91.188 \text{ GeV} , \\
\alpha_{\text{EM}}^{-1}(M_Z) &= 127.9 + \frac{8}{9\pi} \log \frac{M_t}{M_Z} , \\
\sin^2 \theta_W(M_Z) &= 0.2319 - 3.03 \times 10^{-5} T - 8.4 \times 10^{-8} T^2 , \\
T &= M_t/[\text{GeV}] - 165 ,
\end{aligned}
\tag{30}
$$

as given [23, 28], we find

$$
m_\tau(M_Z) = 1.746 \text{ GeV} , \ \alpha_\tau(M_Z) = \frac{g_\tau^2}{4\pi} = 8.005 \times 10^{-6} ,
$$

which, together with α_{EM} and $\sin^2 \theta_W$ given in (30), we use as the input for the RG evolution. In fig. 3, 4 and 5, we show the predictions of model IA on M_t, $m_b(M_b)$ and $\alpha_3(M_Z)$, respectively. Note that the mass values are before the MSSM corrections are taken into account. Since $m_b(M_b)$ agrees with the experimental value $(4.1 - 4.5)$ GeV [23] as we can see from fig. 4, these corrections should be rather small, implying that our model favors the non-universal soft supersymmetry breaking terms [21].

Fig. 6 shows that M_t is relatively insensitive to a change in $\tilde{\alpha}_T$. The reason is that the predicted values for M_t are not very much far from its infrared value [24], which we define as the value for $\tilde{\alpha}_T = 6$ and is shown in fig. 7. By comparing figs. 6 and 7, we see that the predicted values for M_t lie a few GeV below the infrared values. So, if the experimental uncertainty can be reduced to less than that order, there will be a chance to test the Gauge-Yukawa unified model we have proposed here. Note that the present experimental value of M_t is (180 ± 12) [GeV] [29].

Finally, we would like to comment on the difference of the prediction on M_t with and without GYU. As pointed out, the M_t prediction without GYU follows from the requirement of the consistent bottom-tau hierarchy. Under the same assumption made for the RG analysis above, we wish to find the allowed values of $\tilde{\alpha}_T$. We find that the requirement of 4.2 GeV $\lesssim m_b(M_b) \lesssim 4.5$ GeV with $M_{\text{SUSY}} = 500$ GeV fixed, for instance,
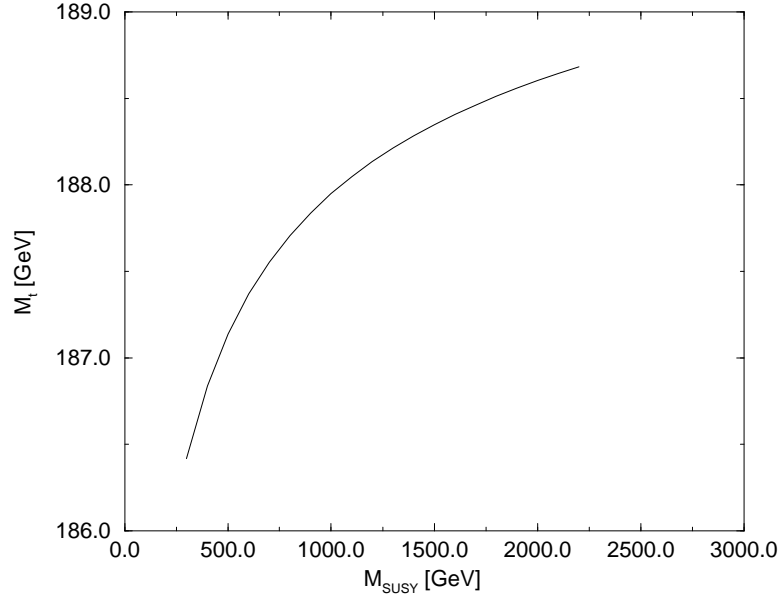
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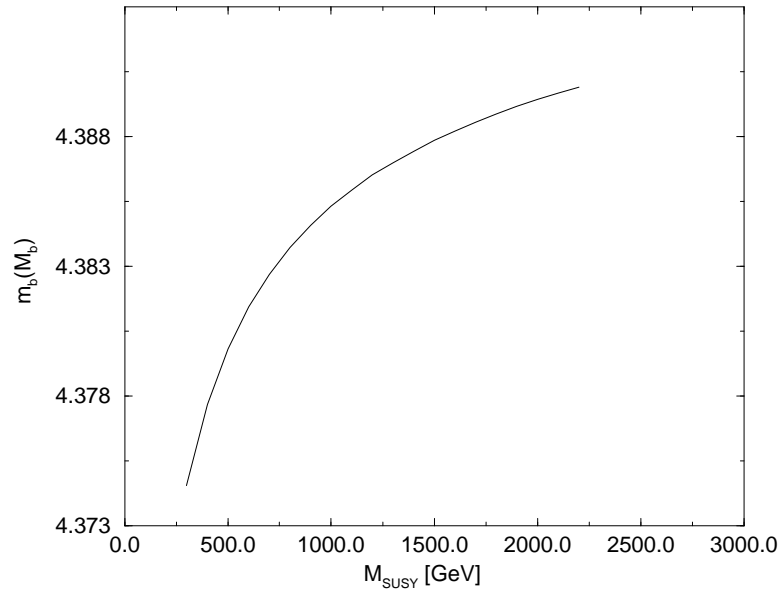Figure 3: M_t prediction versus M_{SUSY} for $\tilde{\alpha}_T = 2.717$.

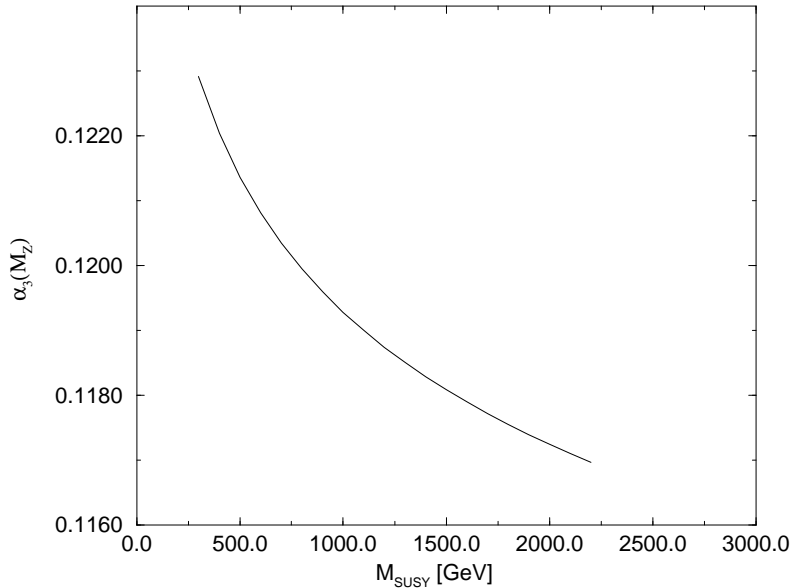Figure 4: $m_b(M_b)$ prediction versus M_{SUSY} for $\tilde{\alpha}_T = 2.717$.

Figure 5: $\alpha_3(M_Z)$ prediction versus M_{SUSY} for $\tilde{\alpha}_T = 2.717$.

implies $1.6 \lesssim \tilde{\alpha}_T \lesssim 7.0$, and consequently, $182.9 \text{ GeV} \lesssim M_t \lesssim 191.2 \text{ GeV}$. This should be compared with the GYU prediction,

$$m_b(M_b) \quad = \quad 4.38 \text{ GeV} , \ M_t = 187.1 \text{ GeV} , \tag{31}$$

which is fixed under the same assumption.

5 Comparison with the infrared-fixed-point approach and discussions

The infrared-fixed-point approach [26] is based on the assumption that infrared fixed points found in first order in perturbation theory persist in higher orders [8] and that the ratio of the compactification scale Λ_C (or the Planck scale M_P) to M_{GUT} is large enough for the ratio of the top Yukawa coupling to the gauge coupling to come very close to its infrared value when running from Λ_C down to M_{GUT}. Since this approach looks similar to

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[8]In the case of the SM, the infrared fixed point approach loses its meaning at the two loop-level [30].
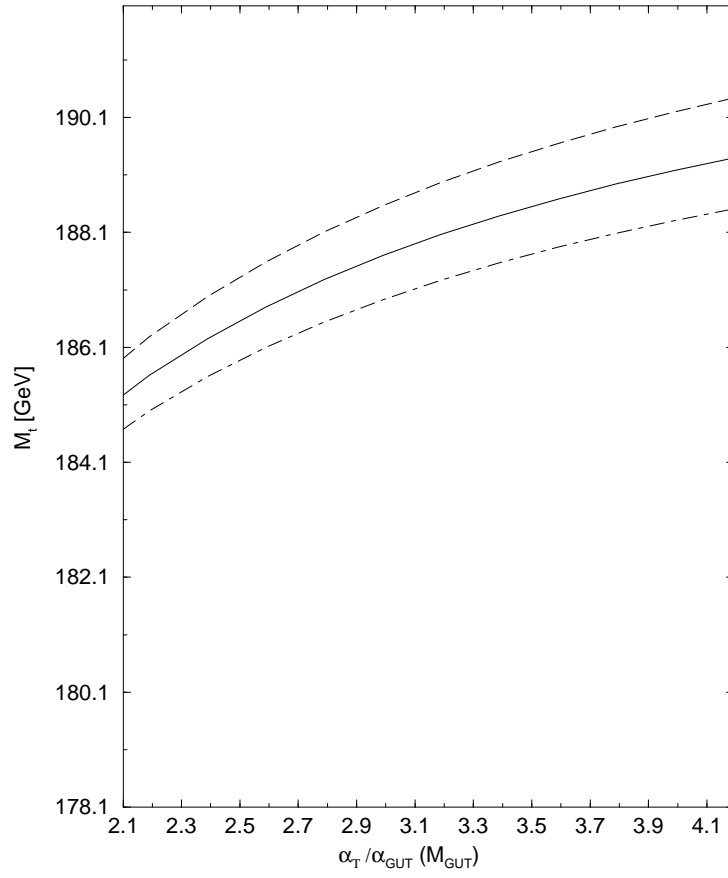
Figure 6: M_t versus $\tilde{\alpha}_T$ with $M_{\mathrm{SUSY}} = 300$ GeV (dod-dashed line), 500 GeV (solid line) and 1 TeV (dashed line).
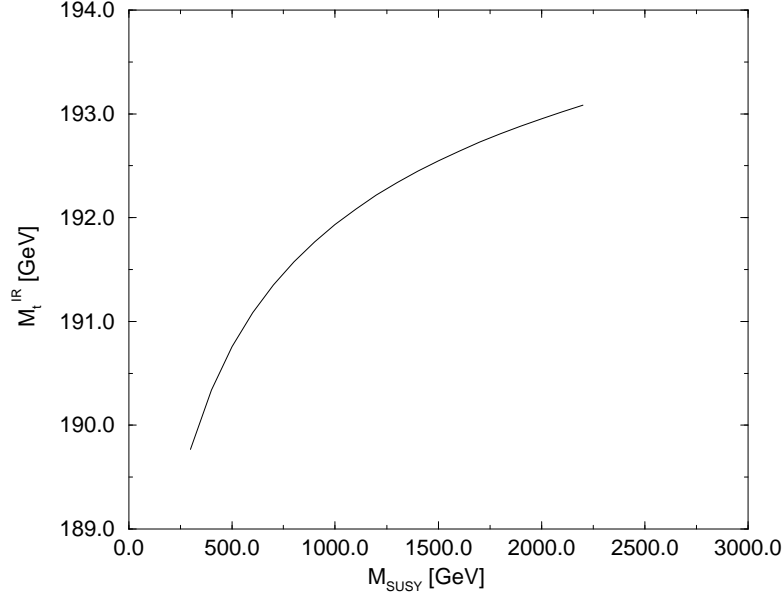
Figure 7: The infrared value of M_t versus M_{SUSY} ($\tilde{\alpha}_T = 6$).

ours at first sight, we would like to examine within the framework of our $SO(10)$ model whether and how much this picture of infrared-fixed-point behavior is realized.

To this end, let us first re-capitulate the argument of ref. [26] and recall that [9]

$$\alpha(\Lambda_{\mathrm{C}}) \;\;=\;\; \frac{\alpha(M_{\mathrm{GUT}})}{1 - (7/2\pi)\alpha(M_{\mathrm{GUT}}) \ln(\Lambda_{\mathrm{C}}/M_{\mathrm{GUT}})} \tag{32}$$

for one-loop order. For $\alpha(M_{\mathrm{GUT}}) = 0.04$ and $\ln(\Lambda_{\mathrm{C}}/M_{\mathrm{GUT}}) = 5$, we obtain $\alpha(\Lambda_{\mathrm{C}}) \simeq 0.051$. Assuming now that α_i's with $i \neq T$ are negligibly small compared with α_T, we derive from the reduction equation (13)

$$\alpha\frac{\tilde{\alpha}_T}{d\alpha} \;\;=\;\; \tilde{\alpha}_T(2\tilde{\alpha}_T - \frac{11}{2}) \;, \tag{33}$$

with the solution [26, 7]

$$\tilde{\alpha}_T(M_{\mathrm{GUT}}) \;\;=\;\; \frac{1}{4/11 + (\kappa_T^{-1} - 4/11)[\alpha(M_{\mathrm{GUT}})/\alpha(\Lambda_{\mathrm{C}})]^{11/2}} \;, \tag{34}$$

where κ_T is the value of $\tilde{\alpha}_T$ at Λ_{C}. The point in the infrared-fixed-point approach is that, since $[\alpha(M_{\mathrm{GUT}})/\alpha(\Lambda_{\mathrm{C}})]^{11/2}$ is small (~ 0.25), the "low-energy" value, $\tilde{\alpha}_T(M_{\mathrm{GUT}})$, is

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[9] Our normalization of the generators of $SO(10)$ differs by a factor of $\sqrt{2}$. Our choice corresponds to that of the usual $SU(5)$ GUTs.
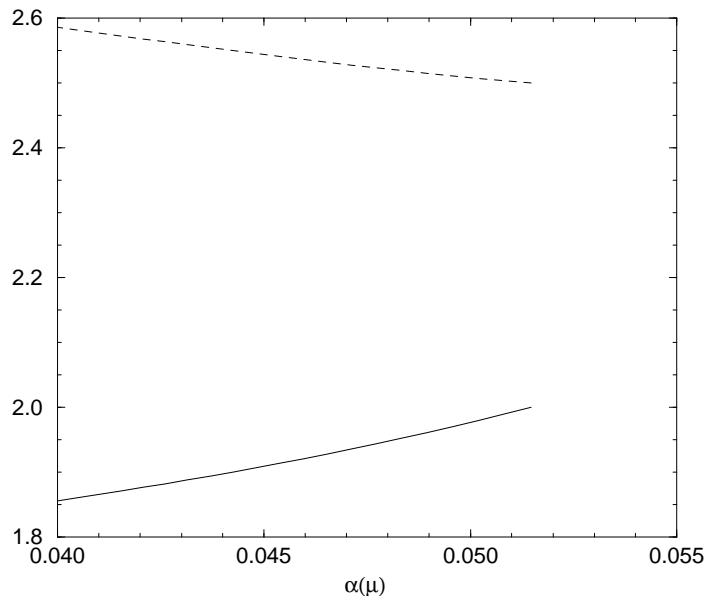
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Figure 8: $\tilde{\alpha}_T(M_{\mathrm{GUT}})$ (solid line) and $\tilde{\alpha}_{HS}(M_{\mathrm{GUT}})$ (dashed line) versus $\alpha(\mu)$ for $\kappa_T = 2$ and $\kappa_{HS} = 2.5$.

insensitive against κ_T and must be close to its infrared value $11/4 = 2.75$. One indeed finds that one may vary κ_T from 1.9 to 4.3 to keep $[\tilde{\alpha}_T(M_{\mathrm{GUT}})/(11/4) - 1] \lesssim 0.1$. It however should be emphasized that there is no principle to fix κ_T in the infrared-fixed-point approach. For $\kappa_T = 1$, which could be realized with the same probability as for $\kappa_T = 2$, we obtain $\tilde{a}_T(M_{\mathrm{GUT}}) \simeq 1.91$, which is only 70% of the infrared value.

The more serious problem is the negligibility of other couplings compared to α_T. Since there exists no reason why the neglected couplings have to be small, they could be large and hence comparable to α_T, thereby changing the infrared structure very much. In fig. 8, we plot $\tilde{\alpha}_T(M_{\mathrm{GUT}})$ as a function of $\alpha(\mu)$ with $0.04 \leq \alpha(\mu) \leq 0.051$, where we have chosen $\kappa_T = 2$ and $\kappa_{HS} = 2.5$ while neglecting the other couplings in the evolution. As we can see from fig. 8, the $\tilde{\alpha}_T$ does not approach to 2.75 as α goes to $\alpha_{\mathrm{GUT}} = 0.04$ from 0.051, rather to another fixed point 1.67. (If κ_{HS} would be zero, the $\tilde{\alpha}_T(M_{\mathrm{GUT}})$ would become 2.51.)

The observation of ref. [26] that, in spite of the small difference between M_{GUT} and Λ_{C}, the Yukawa couplings tend to converge to their fixed points very fast thanks to large

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anomalous dimensions of the matter superfields, is generally correct in one-loop order. However, the infrared-fixed-point approach may not always have predictive power, as we have explicitly seen above in our concrete model. This is not a specific situation of the present $SO(10)$ model, because the factor $[\alpha(M_{\mathrm{GUT}})/\alpha(\Lambda_{\mathrm{C}})]^{11/2} \simeq 0.25$ is small enough according to the discussion of ref. [26]. If one insists from the beginning to choose the one-loop infrared fixed point which is most predictive, the lowest order prediction is exactly the same as that of the reduction of couplings. The difference appears in the next order, because the reduction solution, except for the lowest order, is not a fixed point solution in general.

If the low-energy prediction for either approach is viable as experienced in some cases including the one discussed in this paper, it might indicate some unknown non-perturbative mechanism of unification of couplings such as the dynamical unification of couplings which we suggested in introduction. In either case, the non-perturbative investigation on asymptotically-nonfree, non-abelian gauge theories will be an important issue in future works.

Acknowledgment

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